A sequence of repeated auctions are employed to sell impressions. The goal is to design auctions that allocate the most effective ads for users, and guarantee:

1. extract enough revenue
2. result in a stable market without incurring too much cost to enter the market.

**Auction-based**
- Low revenue: individual auctions too thin, lack of competition.
- Low entering cost: simple rules, allow real-time changes.

**Contract-based**
- High revenue: bundling impressions together.
- High entering cost: not allow real-time changes.

**Dynamic mechanisms - weakness**
- Weakness #1: too complicated.
- Extremely large designing space.
- Even hard to describe the policy.
- Weakness #2: require commitment power.
- Require strong future commitment power.
- Lack of individual rationality.

**Bank Account Mechanisms overcome the weaknesses!**

1. Selecting stage mechanism
   - Seller at each stage selects a stage mechanism \( M_t = (z_t, q_t) \) based on the current balance \( bal_t \) according to the selection policy.

2. Stage mechanisms
   - Each \( M_t \) is a static mechanism — takes report \( v_t \) as input, and outputs allocation \( z_t(v_t) \) and payment \( q_t(v_t) \). Moreover, \( M_t \) itself satisfies IC and IR constraints.

3. Spend policy
   - Seller spends the balance according to the spend policy \( s_t \) and the current balance \( bal_t \). The spent amount, \( s_t(bal_t) \), becomes part of Seller’s revenue. The selection and spend policies are designed to make the entire bank account mechanism IC. The spent amount, \( s_t(bal) \) must be identical (up to a constant) with the expected Buyer utility of \( M_{t+1} \) so that Buyer would be glad to give out its balance and get the same amount of expected utility at the next stage in return.

4. Bank Account Balance
   - The revenue of a bank account mechanism consists of Buyer’s payment in each stage mechanism \( M_t \) and the spends \( s_t \) at each stage.

5. Deposit Policy
   - Buyer deposits money into the bank account. Deposit policy \( \delta_t \) specifies the amount to transfer according to the current balance \( bal_t \) and Buyer’s report \( v_t \).

6. Revenue vs Deficits
   - Spends boost the total revenue, but cause deficits. Limits on balance provide trade-offs between revenue and deficits.

**Example**
- One buyer, two stages, i.i.d. valuations: \( v_1, v_2 \sim F \).
- \( F: \Pr[v = 1] = \Pr[v = 2] = 1/2 \).
- Revenue: Auction-based = 2; Contract-based = 3; Dynamic = 2.5.

**The Dynamic Mechanism and Corresponding BAM**

**Mechanism 1 (Dynamic Mechanism)** Dynamic mechanism achieves revenue 2.5.

- **Stage 1**: select the first item at price 2.5.
- **Stage 2**: allocate the second item if and only if the first item was sold.

**Mechanism 2 (Bank Account Mechanism)** The corresponding bank account mechanism.

- Seller sets the stage mechanism \( M_t \) to be the posted-price auction at posted-price 1.
- Buyer chooses to buy the first item or not. (Buyer reports to the stage mechanism \( M_t \))
- If Buyer buys the first item, deposits 1.5 into the bank account.
- If the balance is 1.5, Seller spends 1.5, otherwise, spends 0.
- If Seller spends a positive amount, sets the stage mechanism \( M_2 \) to be “give for free” mechanism; otherwise, sets \( M_2 \) to be “not for sale” mechanism.
- Buyer gets the second item for free, if \( M_2 \) is “give for free”, and gets nothing if \( M_2 \) is “not for sale”.

**Double-Reserve Auction (DRA)**

In practice, some properties are very important for an auction, such as:
- Deterministic allocations.
- The stage payment should be zero, if nothing gets allocated in that stage.

We then propose the double-reserve auction, which is a subset of BAM and in general does not include any optimal BAM.

**Mechanism 3 (Double-Reserve Auction)** A double-reserve auction, for each stage, has a low reserve price and a high reserve price.

- At each stage, it runs a posted-price auction.
- If the item at the previous stage was sold, the low posted-price for the current stage; otherwise, the high posted-price.

The DRA is extremely simple and easy to describe, and by Theorem 4, the computation of the optimal DRA is not hard.

**Heuristic Double-Reserve Auction (HDR)**

To further simplify the construction, we propose heuristic DRAs (HDR), which are:
- easy to construct — (each stage) only need
  - the Myersen reserve.
  - and at most two queries to the integration oracle.
- nearly optimal for various distributions.

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**Statement of Theorems**

**Theorem 1 (Optimal revenue is achievable by BAMs)** For any direct dynamic mechanism \( M_t \), there is a constructive BAM \( B_t \), such that \( Rev(B_t) \geq Rev(M_t) \).

**Theorem 2 (Revenue and utility guarantee of DRAs)** From any sequence of static posted-price auctions, we can construct the the double-reserve auction \( B_t \) with any given bank account limits, such that comparing with simply running the sequence of static posted-price auctions,
- \( B_t \) has the same buyer utility,
- \( B_t \) has higher revenue, and the revenue is weakly increasing as the bank account limits grow.

**Theorem 3 (Increments come from bundling “future benefits”)** The optimal revenue of a bank account mechanism is bounded by the optimal revenue of static/history-independent mechanisms plus its expected spends, \( \sum_{t \geq 0} E[v_t] \).

**Theorem 4 (Computation of optimal DRAs)** The optimal double-reserve auction could be computed via a dynamic program. In particular, there is an FPTAS for multiplicative revenue approximations.